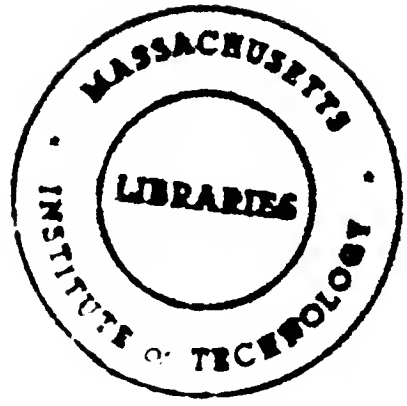
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AN EXPERIMENT IN APPROVAL VOTING

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ABSTRACT

In its 1985 election for council The Institute of Management Sciences (TIMS) ran a test of approval voting by sending a non-binding approval ballot to its members along with the regular plurality ballot. In approval voting a person votes for (approves of) as many candidates as desired, the winner being the candidate with the most votes. Two of the TIMS contests had three candidates for a single office and a third had five candidates for two offices. In such situations approval voting can produce winners who are more generally acceptable to the electorate than standard plurality voting. This is because the approval mechanism tends to prevent two candidates with broad appeal from splitting a majority constituency and electing a minority candidate.

Of these three TIMS contests two would have had different winners had approval voting been binding. In the first election with candidates A, B, and C, C narrowly beat B in the regular election, but B was the approval winner because considerably more of A's supporters approved of B than C. In the second election approval and plurality voting agreed on the winner. In the third election with two positions to be filled from a field of five candidates, A, B, C, D, and E, B was a winner by either method, but the official second winner, A, placed fourth in the approval vote behind C and D.

Close examination of ranking data shows that in the first election a head to head contest between B and C would be a toss up. Nevertheless, because of secondary support, B is a better choice by the criterion of broad acceptance by the electorate. In the third election C and D are similarly close but either would have broader support than the plurality choice A and of the two C has somewhat broader general approval.

It should be pointed out that, as approval voting creates new winners, it must also create new losers. A minority constituency has more difficulty taking advantage of a majority split to install their candidate.

The TIMS experiment demonstrated remarkably well the points made by the proponents of approval voting, namely, that it is a simple and effective way to do a better job than plurality voting at selecting the candidates who have the broadest base of support among the electorate.

1. Introduction

In the spring of 1985 The Institute of Management Sciences (TIMS) conducted a field test of approval voting, an innovative method of assessing the wishes of an electorate. Approval voting is a simple procedure that differs in modest but important ways from regular plurality voting and has certain advantages over it. In the TIMS test all members received an experimental ballot along with the regular official ballot during the annual election of officers and council. Most voters filled in the test ballot.

The experiment was extremely successful in the sense that it showed that approval voting can make a difference and, by example, how those differences can occur and what they signify. In the TIMS election several outcomes would have changed had the approval ballot been binding and, equally as interesting, several others would not. Our analysis shows that the changes would have resulted in electing candidates that had a broader base of popular support than those actually elected.

Voting plays a central role in the democratic process as a means of selecting individuals to hold positions of responsibility. We believe that approval voting provides a simple mechanism that, in cases where it makes any difference, will select candidates who are more widely desired by the electorate than standard plurality voting. By the same token, however, approval voting makes it more difficult for a small organized group to take advantage of a split majority and elect a minority candidate.

All candidates in the TIMS election were informed of the pending experiment by H. Newton Garber, chairman of the 1984-5 nominating committee, and agreed to participate. The society is grateful for their cooperation. To preserve anonymity in so far as possible, the candidates will be identified only by code designations (A,B,...). In addition the elections will not be identified by office.

Section 2 of the report describes approval voting. Section 3 summarizes the experimental design and the basis of the analyses. Sections 4, 5 and 6 analyze the data for three elections. The final section summarizes the findings. Additional supporting data appear in the Appendix.

2. Approval Voting

Most elections, including those of TIMS, are conducted by the familiar method of plurality voting. When one person is to be elected, voters are instructed to vote for one from a list of candidates. When more than one position is to be filled from a single list, voters are asked to vote for as many people as there are positions to be filled. The candidates receiving the most votes are elected.

Approval voting differs from plurality voting in one important way. Instead of restricting the number of candidates a person may vote for in an election, approval voting allows each person to vote for any number. This gives a person the opportunity to vote for every candidate he or she finds acceptable or approves of, or to discriminate between a more preferred subset of candidates and the rest. Each candidate selected receives a full vote. The candidates receiving the most votes are elected.

Brams and Fishburn (1983) have analyzed approval voting in depth and argue that it is superior to plurality and other simple election procedures when there are three or more candidates to choose from. Their basic thesis is that approval voting is an efficient and effective way to assess the overall support for each candidate within the electorate, and that elected candidates should be those who are most widely supported or approved of. Analyses of past elections conducted essentially by plurality voting, including multi-candidate contests for the United States Senate and Presidency, clearly show that this is not always so under plurality voting. For example, each of the 1970 and 1980 New York

Senatorial elections had three main candidates on the ballot. Each election had a majority candidate, i.e., one who would have defeated each of the other two in simple pairwise contests. Yet in both cases the majority candidates lost the plurality elections because some of their support was diverted to rather similar candidates, thereby permitting minority candidates to win. If approval voting had been used, the majority candidates would have been elected.

A further introduction to approval voting appears in Brams and Fishburn (1983). In practice, approval voting would use the same type of ballot as plurality uses and would be only slightly harder to tabulate. More complex methods, including ones that ask voters to rank order candidates, require substantially more processing, and it is not clear that such methods would do better at uncovering the candidates most favored by the electorate.

3. The TIMS Experiment

The experiment was announced to TIMS members in John D. C. Little's presidential column in the February 1985 issue of OR/MS TODAY. On April 21, two ballots were mailed to each TIMS member. The official plurality ballot determined the binding results of the election. The experimental ballot assessed the effects of approval voting in the three races with at least three candidates. We shall designate these three elections as:

E1: an election of one out of three candidates;
E2: another election of one out of three;
E3: an election of two out of five candidates.

In these elections each voter was asked to check approved candidates to the right of the names on the experimental ballot and to rank the candidates (1,2,...) to the left of the names. Although rankings are not part of approval voting, they help to determine majority comparisons. Since majority candidates (which might not exist if there are cyclical

majorities) are often claimed to be most desirable, it was felt important to include this information.

The official ballot was returned by 1,851 members. Of these, 1,579 (85%) also returned the test ballot. This remarkably high rate of return shows widespread cooperation by the members and lends credence to the results. Further support arises in the exceptionally small number of inconsistencies: see the Appendix for details.

Almost all the test ballots had some approvals checked, but a few did not rank the candidates. The counts by response category were:

R1.	Only official ballot returned:	272	(14.7%)
R2.	Test ballot also returned, but without rankings:	68	(3.7%)
R3.	Test ballot also returned with some rankings:	1511	(81.6%)
		1851	

Slightly different numbers of voters participated in each election, as will be reflected in the ensuing analyses.

We discuss each of the three elections in turn in the following sections. Three aspects of each are presented: official plurality counts, approval voting counts, and majority comparisons.

4. Election E1

In election E1, a three-way race with candidates A, B, and C, approval voting would have changed the winner from C to B. See Table 1. Candidate C won the official election by 8 votes (0.4% of the votes cast) but, had approval voting been in effect, B would have won by 130 (5.5% of the votes cast) using the actual returned approval votes. B would have won by 170 (6.1%) in extrapolated totals.

In Table 1 the extrapolated totals take account of the people who submitted an official

TABLE 1

Official and approval votes for election E1 show that C won the official election but B would have won under approval voting

	Official plurality vote	Actual approval vote	Extrapolated approval vote
A	166	417	486
B	827	1038**	1224**
C	835**	908	1054
	1828	2363 (1567 voters)	2764 (1828 voters)

** largest vote

ballot but not the test ballot. This is done by assuming that the non-responders would have behaved in the manner of the responders to both ballots. Let

N_X = number of non-responders who voted for candidate X on the official ballot;

$P(Y|X)$ = fraction of responders approving candidate Y given that they voted for X on the official ballot.

Then our estimate of additional approval votes attributable to non-responders is:

$$\text{added approval votes for } Y = P(Y|A)N_A + P(Y|B)N_B + P(Y|C)N_C .$$

We see from Table 1 that the extrapolation does not change the results in any fundamental way.

Next we look for a majority candidate. As stated earlier, a majority candidate is one who would win in pairwise (plurality) elections with all other candidates. The analysis here is a little more complicated and makes use of the rankings. Table 2 presents the results.

TABLE 2

For election E1, majority comparisons show that in hypothetical two-way races both B and C would defeat A but that B against C would be a virtual tie

Contest:	A vs. B		A vs. C		B vs. C	
Inferred votes for:	A	B	A	C	B	C
Revealed first choices from official ballot counts:	166	827	166	835	827	835
Revealed second choices from:						
R3 rankings	166	492	290	331	70	66
R2 approvals	2	3	4	9	3	0
Subtotal of revealed choices:	334	1322	460	1175	900	901
Estimated second choices:	44	128	89	104	14.2	12.8
Extrapolated total votes:	378	1450**	549	1279**	914.2**	913.8

** larger vote

We see from the totals that B would beat A, C would beat A, and that B and C are virtually in a dead heat. The analysis that leads to these results will be illustrated with the B versus C comparison. 827 voted for B on the official ballot and are listed in the first row as preferring B to C (and also B to A). Similarly 835 prefer C to B. However, 166 people cast votes for A. How would they vote in a B versus C election? The ballots of these 166 show that

70 provided rankings in the order ABC,
66 provided rankings ACB,
3 provided no rankings but approved A and B,
27 made no distinction between B and C by ranking or approvals.

Thus in a B versus C comparison the 70 are credited to B, the 66 to C, and the 3 to B.

This leaves 27 who voted for A but gave no indication of a B versus C preference. If the 27 are ignored, the revealed choices are 900 for B and 901 for C. The next to last row of Table 2 estimates how the 27 would split between B and C if they adhered to the pattern of the 139 voters (70 + 66 + 3) who voted for A and also expressed a preference between B and C. The calculation for B is $27(73/139) = 14.2$ and for C is $27(66/139) = 12.8$. The final totals, carried into decimals, are 914.2 for B and 913.8 for C. We declare ourselves unable to make a valid distinction between the two candidates.

A few further notes on E1: The distribution of number of approvals among the 1435 voters who approved one or more candidates was:

1 approval:	660	(46%)
2 approvals:	706	(49%)
3 approvals:	69	(5%).

Brams and Fishburn (1983) observe that votes for either one or two candidates in a three-candidate election are equally efficacious, i.e., have the same chance of affecting the outcome. In this particular test election, the voters who approved of one or two candidates were fairly evenly split. Three approvals have the same effect on the outcome of the election as not voting. However, three approvals do indicate that all the candidates are acceptable to the voter as people to perform the functions of the office.

To summarize the results from election E1, C wins the official plurality election by 0.5%, B would have won an approval vote by 6.1% and a head-to-head election would be too close for us to call. The picture emerges that C has a loyal following that is just a little bit larger than B's. However, among A's followers, more approve of B than C (36% to 23%). Furthermore, more of C's followers approve of B (43%) than B's followers do of C (27%). Hence B wins the most approval votes. What can we conclude? Although we

cannot definitively say who would win a head-to-head election between B and C, it appears that B has a broader acceptance in the electorate.

5. Election E2

Election E2 is another three-way race to elect one person, but here plurality and approval results agree. Since once again the extrapolated approval votes do not change the fundamental outcome, we omit them for this case. Table 3 presents the results.

TABLE 3

In election E2 plurality and approval votes pick the same winner

		Official plurality vote	Actual approval vote
Candidate	A	395	590
	B	552	752
	C	597**	793**
		1544	2135 (1354 voters)

** largest vote

Table 3 also demonstrates an attractive feature of approval voting for three-candidate contests in which each candidate polls a sizable plurality. The winning plurality is 38.7%, well below 50%, whereas the largest approval count is 58.6%, comfortably over 50% of the voters. Hence even though C won with only a 39% plurality, C was approved of by a majority of the voters.

Table 4 shows that candidate C is not quite a majority candidate. C defeats A (as does

B) in pairwise races, but C and B are in a virtual tie when run against each other. Such a closeness would not have been anticipated from Table 3 and arises because A's supporters tend to prefer B to C.

TABLE 4

For election E2, C is not quite a majority candidate: in hypothetical two-way races B would defeat A, C would defeat A, but C versus B is too close to call

Contest:	A vs. B		A vs. C		B vs. C	
Inferred votes for:	A	B	A	C	B	C
Revealed first choices from official ballot counts:	395	552	395	597	552	597
Revealed second choices from:						
R3 rankings	171	281	179	240	162	128
R2 approvals	1	3	2	1	0	1
Subtotal of revealed choices:	567	836	576	838	714	726
Estimated second choices:	53	88	56	74	58	46
Extrapolated total votes:	620	924**	632	912**	772	772

** larger vote

The distribution of the 1286 voters who approved of one or more candidates on the test ballot is:

1 approval:	691	(54%)
2 approvals:	421	(33%)
3 approvals:	174	(13%).

In summary election E2 presents an example in which the plurality and approval votes agree on C. We cannot say that C would actually beat B in a two-way race, but, as in E1, the approval vote indicates a wider acceptance of C.

6. Election E3

A new surprise awaits us in election E3, in which two people are to be elected from among 5 candidates, A, B, C, D and E. We find that the second place winner in the official plurality election finished fourth in the approval voting. Table 5 shows the summary. As in election E2, the extrapolated approval votes show the same pattern as the actual and are omitted.

TABLE 5

In election E3 with two positions to be filled, the first choice is candidate B under both methods but the second choice by plurality voting, candidate A, drops to fourth place behind C and D in approval voting

		Official plurality vote	Actual approval vote
Candidate	A	679*	669
	B	937**	956**
	C	651	715*
	D	670	685
	E	391	483
		3328 (1749 voters)	3508 (1380 voters)

** largest vote

* second largest vote

The majority comparisons of hypothetical pairwise contests between the candidates are shown in Table 6.

TABLE 6

Results of hypothetical pairwise contests between the five candidates in E3 show that approval winners B and C would win pairwise contests with all others

Contest	Votes inferred for	Revealed first choices from rankings	Other revealed choices from rankings	Revealed choices from approval ballots	Totals
A vs B	A	337	203	10	550
	B	443	376	25	844**
A vs C	A	337	312	16	665
	C	266	420	22	708**
A vs D	A	337	303	12	652
	D	259	393	21	673**
A vs E	A	337	398	15	750**
	E	103	394	10	507
B vs C	B	443	392	19	854**
	C	266	234	13	513
B vs D	B	443	389	22	854**
	D	259	260	17	536
B vs E	B	443	537	26	1006**
	E	103	250	6	359
C vs D	C	266	420	15	701**
	D	259	348	17	624
C vs E	C	266	535	22	823**
	E	103	364	9	476
D vs E	D	259	520	24	803**
	E	103	375	9	487

** larger vote of pair

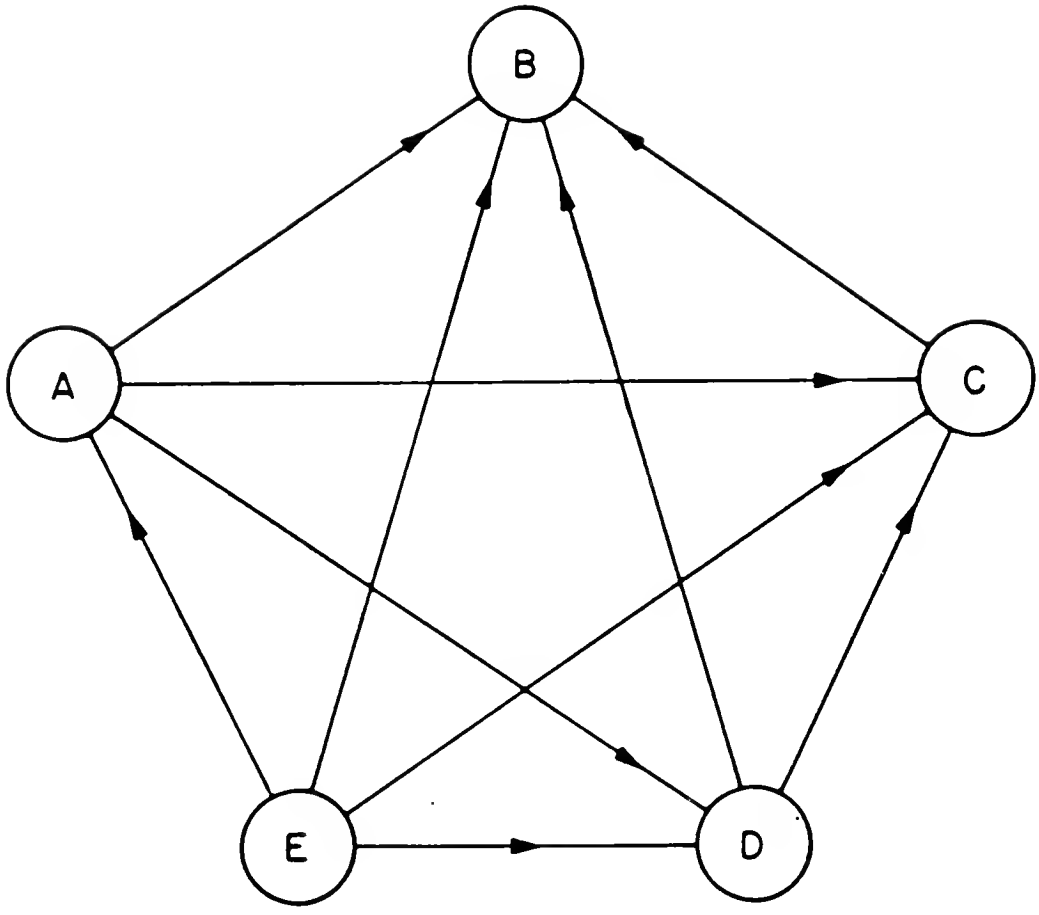

FIGURE 1. A graph of the hypothetical pairwise contests within E3 with arrows toward winners. The two winners are B, then C, and since there are no directed cycles, no cyclical majorities are present.

Table 6 and Figure 1 show that the majority comparisons for E3 are transitive with ranking B C D A E. The rankings from the three types of analysis are:

plurality:	B A D C E
approval:	B C D A E
majority:	B C D A E.

The approval ranking is identical to the majority ranking. On the other hand, as already noted, the plurality ranking elevates the fourth place majority candidate to second place

and drops the majority runner-up C to fourth place.

What has happened in E3 is that B clearly has wide support. Beyond that a cohesive minority has voted for A. However, when the electorate is given an opportunity to express their opinions by taking off the two vote constraint, many more approve of C and D than A. For example, 23% of those not voting for C on the official ballot approved of C. Similarly, 20% not voting for D approved of D. But only 14% of those not voting for A approved of A.

In other information from E3 the distribution of the 1380 voters who approved of at least one candidate is

1 approval:	116	(8%)
2 approvals:	641	(46%)
3 approvals:	494	(36%)
4 approvals:	75	(5%)
5 approvals:	54	(4%).

Thus most voters approved of either two or three candidates.

7. Conclusions

Our analysis indicates that approval voting captures the general wishes of the electorate considerably better than plurality voting. The approval method compares favorably with rankings methods, as shown for example by majority comparisons, but it requires less effort of the voter and tabulator and is also much less vulnerable to strategic misrepresentation by voters than are ranking methods (Brams and Fishburn, 1983). A possible detraction is that a determined minority constituency may be unable to take advantage of a split majority to elect its candidate but instead will have to seek approval from the whole electorate.

Our work with the experimental data has led us to believe that approval voting would be desirable for TIMS. At the same time, consideration should be given to partition

multiple-seat elections into multiple single-position elections when it is desired to represent particular constituencies. This is presently done, in part, by separating U.S. and non-U.S. Council seats.[1]

1 We are deeply indebted to Newton Garber for his central role in the TIMS approval voting experiment and to Mary DeMelim and the TIMS business office for their help in designing and counting ballots.

Appendix

We provide here various further data that support and enrich the analyses in the text, especially for elections E1 and E3.

Ballot transcription and discrepancies

The TIMS Business Office transcribed the official ballot votes onto the test ballot for each voter who returned both to provide consistency checks and to link the two ballots. A few minor discrepancies arose from transcription. For example, the official plurality votes in E1 for A, B and C are 166, 827 and 835 respectively (Table 1). The same counts from the transcriptions on the test ballots are 167, 826 and 835 respectively (see Table 7 "official plurality vote"). None of the discrepancies affects our conclusions.

Several interesting inconsistencies occurred in the voting. Nine voters in election E3 approved of a candidate ranked lower than one they did not approve of. Two E1 voters did not approve of their official ballot choice, yet approved of some other candidate. Seven voters did the same in E2. In the five candidate E3 election, 18 voters failed to approve of a candidate selected on their official ballot yet approved others not selected on the official ballot.

Rankings

In election E1, 95% of the voters who ranked one or more candidates on the test ballot ranked all three (1415 of 1484). In E2 it was 88% (1161 of 1313). In E3, 1420 voters ranked the candidates, of whom 46 ranked only their first choice, 210 ranked their first two choices, 189 ranked three, and 975 (69%) ranked all five. A few voters indicated ties in their rankings.

Approval voting, E1

Table 7 shows the match-ups between approval votes and official plurality along with other data used to obtain the approval vote summaries in Table 1. The upper part of

TABLE 7

Analysis of approval voting and plurality voting for election E1 shows approval voting sets matched with official plurality votes along with summaries and extrapolations

Candidates in approval set (R2 + R3)	Voter's choice on official ballot A	B	C	none	Total ballots
none	4	18	46	0	68
A	66	0	0	0	66
B	2	345	1	2	350
C	0	0	289	1	290
AB	41	133	1	0	175
AC	24	0	79	2	105
BC	0	174	268	0	442
ABC	9	22	40	0	71
Subtotals	146	692	724	5	1567
Approval votes for:					Actual approval vote (sum)
A	140	155	120	2	417
B	52	674	310	2	1038
C	33	196	676	3	908
Other official votes (R1):	21	134	111		266
Official plurality vote:	167	826	835		1828
Extrapolated additions to approval vote for:					Extrapolated approval vote total
A	21(140/146) + 134(155/692) + 111(120/724)		=	69	486
B	21(52/146) + 134(674/692) + 111(310/724)		=	186	1224
C	21(33/146) + 134(196/692) + 111(676/724)		=	146	1054

the table includes all 1567 voters who returned a test ballot for E1 along with their official ballot. For example, of the 105 voters who approved of A and C, 24 voted for A on the official ballot, none voted for B, 79 voted for C, and two voted for nobody. The "actual approval vote" column in Table 1 is obtained from the totals column in the upper part of Table 7.

The middle part of Table 7 summarizes the number of approval votes for each candidate as a function of the candidate voted for on the official ballot. The next two lines record the test-ballot nonresponders who voted on the official ballot ($N_A = 21, N_B = 134, N_C = 111$), and the total plurality votes (counted from transcriptions on the test ballots plus the nonresponders totals).

The bottom section shows the calculation for extrapolated approval vote totals that was described in Section 4.

Approval voting, E3

Table 8 includes the official votes for election E3 for each of the ten two-candidate subsets, matched against the subsets approved by these voters. For example, row ABC shows that 23 people who voted for A and B on the official ballot, 20 people who voted for A and C, 16 people who voted for B and C, and one person who voted for C and E all approved of A, B and C on the test ballot. Contrary to instruction on the official ballot, a number of people voted for only one of the five candidates on that ballot. These votes were counted in the official totals but are not reflected in Table 8.

Summaries of the data in the main table are given at the bottom. Conditional relative frequencies for approving of a candidate not in the official ballot pair can be computed from the table. For example, of the 255 voters who voted for B and C on the main ballot,

TABLE 8

Approval votes for election E3 are matched with candidate pairs voted for on the official ballot

		Candidate pair voted for on official ballot											
		AB	AC	AD	AE	BC	BD	BE	CD	CE	DE	Total	
	A	6	2	1								9	
	B	5				7	4	2				18	
	C		2			2			4	3		11	
	D			1					4		2	7	
	E				2			1			3	6	: 1 A. V. = 51
	AB	99	1			1	1					102	
Approval	AC		34									34	
set	AD	1		67	1							69	
	AE			1	46							47	
	BC					137	1	1				139	
	BD						83					83	
	BE					1		47				48	
	CD			1		1	2		63		1	68	
	CE									31		31	
	DE										24	24	: 2 A. V. = 645
	ABC	23	20			16				1		60	
	ABD	30		22			12					64	
	ABE	28			24			14				66	
	ACD		8	15			1		8	1		33	
	ACE	1	7		9					1		18	
	ADE			18	7						7	32	
	BCD					38	35		32			105	
	BCE					31		17	1	13		62	
	BDE						14	10			5	29	
	CDE								5	2	2	9	: 3 A. V. = 478
	ABCD	8	4	3			3		2			20	
	ABCE	3	1		3	3		2				12	
	ABDE	2		2	2		2				1	9	
	ACDE			2	2							4	
	BCDE					9	6	5	2	4	1	27	: 4 A. V. = 72
	ABCDE	6	1	9	3	9	10	3	8	1		50	: 5 A. V. = 50
	Total:	212	80	142	99	255	174	102	129	57	46	1296	

	official ballot count above	ballots with approval
A	533	629
B	743	894
C	521	683
D	491	633
E	304	474
	2592 (= 2 × 1296)	3313 (= 51 + 2 × 645 + 3 × 478 + 4 × 72 + 5 × 50)

29 approved of A (11%), 57 approved of D (22%), and 53 approved of E (21%). As noted in Section 6. 14% of the voters who did not vote for A approved of A, 23% who did not vote for C approved of C, and 20% who did not vote for D approved of D.

Reference

Brams, S. J. and P. C. Fishburn, *Approval Voting*, Birkhäuser, Boston, 1983.